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Bifurcation memory in mathematical physics of biological objects: the current state of research

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"Bifurcation memory" (BM) is a generalized name for some specific features of behaviour of the dynamical system near the bifurcation. The essence of the behaviour lies in the appearance of a special type of transition process. A review of the results achieved in researches of the BM and a discussion of prospects such researches are given.

1 The definition

The term "bifurcation memory" (BM) is commonly understood as an unusual transient process, sometimes observed in the behaviour of a system located near the bifurcation boundary, namely in the region of the "bifurcation spot" [1–3].

2 Introductory remarks on the current state of research of biological objects using methods of physics

The distance between such scientific subdivisions as "*the physics of biological objects*" (e.g. "cardiophysics") and "*physics in biological science*" (e.g. "biophysics of the heart") is gradually and inevitably reduced, but at present remains very noticeable. The first of these implies a scientific consideration of biological objects within the framework of the paradigm adopted in physics; the second should be understood in the narrow sense as a set of instrumental methods borrowed from physics and used within the framework of the paradigm that has developed in biological science.

2 Review of the results achieved in the bifurcation memory researches

In view of the foregoing, it should be recognized that the phenomena of bifurcation memory now continue to remain largely the subject of mathematical physics in general — and the mathematical physics of biological objects in particular — but they are still understood and perceived very weakly by the representatives of biophysics.

The situation has set partly due to the fact that the experimental confirmation of the existence of BM in biological objects has so far been reliably obtained only in one work [2], quite fresh.

At the same time, a number of predictions related to BM have recently been given within the framework of cardiophysics, which follow quite clearly from the analysis of mathematical models, but have not yet been confirmed in the natural experiment: 1) the existence of "tachycardia of the lacetic type" [4], 2) special forms of action potential for pacemaker activity [5], 3) bifurcation mechanism of trigger activity [6]. These results of theoretical studies are waiting for their experimental verification.

Let us also recall the fact that the main results in BM research were obtained when considering the problems of ensuring safety in transport [1], and it was shown that the vehicle, being in the BM zone, loses its controllability.

On the other hand, it was shown [2] for the biological system (namely, for the blood coagulation system) that its staying exactly in the BM region corresponds to its normal behaviour, and any deviation of this system from the BM region leads to the death of the biological organism as a whole.

3 Review of the prospects for researches on bifurcation memory

3.1 The significance of staying in the area of a bifurcation spot for biological systems compared to technical systems

Summarizing the above scant results, we can note a fundamental difference in the role of BM for technical systems and for biological systems, but the question remains how far such conclusions can be extended. As to technical systems, the situation seems to be clearer, since these systems are intentionally created by people in such a way that they, in their operating mode, must be far from the bifurcation boundaries. However, the question, whether existence in the interior of the bifurcation spot is characteristic for biological systems, requires wide-ranging research.

Taking into account the influence of BM on the controllability of vehicles, one should conclude that the problem of ensuring safety in transport requires carrying out some detached researches.

From a practical point of view, it is important in either case to develop methods for an accurate quantitative analysis of BM with the aim of finding out the width of the boundary zone in which the phenomenon of BM exists, i.e. the exact location of the "bifurcation spot". Some approaches to the solution of this problem were proposed earlier [7].

2.2 Bifurcation memory in reaction-diffusion systems

For many decades, the subject of close attention of researchers has been systems of reaction-diffusion type (RDS), and that is due to their widespread among the natural objects that are commonly referred to as *active media*. It were the RDSs, where a wide class of processes, known as "*autowave processes*" [8], was discovered. The most common application of RDS at the present time is modelling of the myocardium, which is connected with the research of autowave processes in the heart; the most recent surveys of such studies are given in [4, 6]. In one of such models of the myocardium, namely, in the Aliev-Panfilov model [9], the phenomena of BM were detected [3].

Whether the phenomena of BM are inherent in all RDSs or are a specific feature of the Aliev-Panfilov model itself, which has a significant non-linearity, remains questionable, and it is obvious that the answer to this question is of fundamental importance for mathematics. In connection with these circumstances, it is relevant to search for BM phenomena in such a seemingly well-studied model as the FitzHugh-Nagumo system [10]. At the same time, the search for BM in widely used models of the myocardium is also very relevant, as these models are widely used for the needs of clinical medicine.

2.3 Experimental proof of bifurcation memory

Verification of the BM in a real natural scientific experiment is connected with certain difficulties caused by the fact that real objects are usually non-homogeneous (both heterogeneous and anisotropic), which masks or distorts the phenomena of BM.

The chemical Belousov–Zhabotinsky reaction, being a classic and most fully studied example of the reaction-diffusion systems in the real world, seems very suitable for such verification, since it allows us to reproduce the conditions of a homogeneous active medium, and there were developed a number of mathematical models of it, the most accurate of which was called "Pushchinator" [11]. If a more complete study of the this model will reveal those areas in the parameter space of the Pushchinator that correspond to the location of the bifurcation spot, this will contribute to the success of the corresponding natural experiments.

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